

# Neutrino Mixing and Leptogenesis in Type-II Seesaw Scenarios with Left-Right Symmetry

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## Abstract

We propose two Type-II seesaw scenarios for the neutrino mass matrix in the left-right symmetric model, in which the Higgs triplet Yukawa coupling matrix takes the appealing Friedberg-Lee texture. We show that the nearly tri-bimaximal neutrino mixing pattern, which is especially favored by current neutrino oscillation data, can be obtained from both scenarios. We also show that the cosmological baryon number asymmetry can naturally be interpreted in these two scenarios via the flavor-independent leptogenesis mechanism.

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## I. INTRODUCTION

Current solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino oscillation experiments have provided us with very convincing evidence that neutrinos have non-vanishing rest masses and their mixing involves two large angles ( $\theta_{12} \sim 34^\circ$  and  $\theta_{23} \sim 45^\circ$ ) and one small angle ( $\theta_{13} < 10^\circ$ ). These important results indicate that the standard electroweak model, in which the gauge group is  $SU(2)_L \times U(1)_Y$  and three neutrinos are massless Weyl particles, is actually incomplete. There are many possibilities of extending the standard model to accommodate massive neutrinos and to resolve or soften other potential problems of the model itself [5]. One of them, motivated by the conjecture that parity is a perfect symmetry at high-energy scales and is spontaneously broken at low-energy scales, is the left-right symmetric model [6]. The elegance of this model and its testability at the LHC and ILC experiments have recently been iterated [7].

The left-right symmetric model is based on the gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , and thus it naturally contains both left-handed and right-handed neutrinos together with one Higgs bi-doublet  $\Phi$  and two Higgs triplets  $\Delta_{L,R}$  [6]. In addition to the left-right gauge symmetry, a discrete left-right symmetry may be introduced into the model by requiring its invariance under  $l_L \leftrightarrow (l_R)^c$ ,  $q_L \leftrightarrow (q_R)^c$ ,  $\Delta_L \leftrightarrow \Delta_R$  and  $\Phi \leftrightarrow \Phi^T$  [8]. The gauge-invariant Yukawa interactions between the fermion and Higgs sectors can be written as

$$\begin{aligned}
-\mathcal{L}_Y = & \overline{q}_L \Phi Y_q q_R + \overline{q}_L \tilde{\Phi} \tilde{Y}_q q_R + \overline{l}_L \Phi Y_l l_R + \overline{l}_L \tilde{\Phi} \tilde{Y}_l l_R \\
& + \frac{1}{2} \left[ \overline{l}_L i\tau_2 \Delta_L \mathcal{F}(l_L)^c + (\overline{l}_R)^c i\tau_2 \Delta_R \mathcal{F} l_R \right] + \text{h.c.} ,
\end{aligned} \tag{1}$$

where  $\tilde{\Phi} \equiv \tau_2 \Phi^* \tau_2$  and  $(l_{L,R})^c \equiv C \overline{l_{L,R}}^T$  with  $C$  being the charge-conjugation matrix. As a consequence of the discrete left-right symmetry,  $Y_{l,q}$  and  $\tilde{Y}_{l,q}$  are both symmetric matrices. Note that  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  is spontaneously broken into the standard-model gauge group  $SU(2)_L \times U(1)_Y$  via the vacuum expectation value (vev) of  $\Delta_R$ , and then the spontaneous electroweak symmetry breaking is accomplished through the vev of  $\Phi$ . Given the vevs of  $\Delta_L$ ,  $\Delta_R$  and  $\Phi$  [6],

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} , \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} , \quad \langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix} , \tag{2}$$

the up-type quark, down-type quark, charged lepton and Dirac neutrino mass matrices are

$$\begin{aligned}
M_u &= \kappa Y_q + \kappa' e^{-i\alpha} \tilde{Y}_q , \\
M_d &= \kappa \tilde{Y}_q + \kappa' e^{+i\alpha} Y_q , \\
M_e &= \kappa \tilde{Y}_l + \kappa' e^{+i\alpha} Y_l , \\
M_D &= \kappa Y_l + \kappa' e^{-i\alpha} \tilde{Y}_l .
\end{aligned} \tag{3}$$

Meanwhile, the left- and right-handed Majorana neutrino mass matrices can be obtained from the corresponding mass terms in Eq. (1) once the Higgs triplets  $\Delta_L$  and  $\Delta_R$  acquire their vevs:  $M_L = v_L \mathcal{F}$  and  $M_R = v_R \mathcal{F}$ . Integrating out the heavy particles (i.e., the right-handed Majorana neutrinos and the Higgs triplets), one obtains the effective mass matrix for three light (left-handed) Majorana neutrinos via the Type-II seesaw relation [9]:

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T = v_L \mathcal{F} - \frac{1}{v_R} M_D \mathcal{F}^{-1} M_D^T. \quad (4)$$

The phenomenon of lepton flavor mixing, which has clearly shown up in both solar and atmospheric neutrino oscillations, arises from the mismatch between the diagonalizations of  $M_e$  and  $M_\nu$ . On the other hand, it is possible to interpret the cosmological baryon number asymmetry  $\eta_B \equiv n_B/n_\gamma = (6.1 \pm 0.2) \times 10^{-10}$  [10] with the help of the thermal leptogenesis mechanism [11]: either through the out-of-equilibrium decay of the lightest right-handed Majorana neutrino, or via the out-of-equilibrium decay of one or more Higgs triplets [12], or due to both effects. The left-right symmetric model is therefore an intriguing playground to explore new physics beyond the standard model, at least in the neutrino sector.

However, it is a highly non-trivial task to simultaneously account for the cosmological baryon number asymmetry and current neutrino oscillation data in the left-right symmetric model. The reason is simply that the specific textures of  $M_e$ ,  $M_D$  and  $\mathcal{F}$  are not fixed by the model itself. To get around this difficulty, one may impose certain flavor symmetries or empirical assumptions on those mass matrices such that their textures can be (partly) determined or constrained. Such a phenomenological strategy has been adopted in some recent attempts [13,14] to study neutrino mixing and leptogenesis based on the left-right symmetry, although not all of them are successful in fitting the updated observational [10] and experimental [15] data.

The purpose of this paper is to propose two novel and viable scenarios for  $M_D$  and  $\mathcal{F}$  in the flavor basis where  $M_e$  is diagonal, so as to simultaneously interpret the observed neutrino mixing pattern and the observed baryon number asymmetry of the Universe in the left-right symmetric model. A salient feature of both scenarios is that the Higgs triplet Yukawa coupling matrix takes the Friedberg-Lee (FL) texture [16–18],

$$\mathcal{F} = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix} + dI, \quad (5)$$

where  $I$  denotes the  $3 \times 3$  identity matrix. Such a texture is phenomenologically appealing for two simple reasons: (1)  $\mathcal{F}$  can be diagonalized by a unitarity transformation whose form is very close to the interesting tri-bimaximal mixing pattern [19]; and (2) the inverse matrix of  $\mathcal{F}$ , which appears in the seesaw formula, has a structure exactly parallel to  $\mathcal{F}$ . It is therefore possible to obtain the nearly tri-bimaximal neutrino mixing matrix, which is particularly favored by current neutrino oscillation data, from the Type-II seesaw relation under a suitable condition. The complex phases of  $\mathcal{F}$  turn out to be the common source of CP violation for neutrino oscillations and baryogenesis via leptogenesis, if the Dirac neutrino mass matrix  $M_D$  is real. To be specific, we shall assume that  $M_D = M_e$  is diagonal and real in scenario (A), and  $M_D$  is real and has a similar FL texture in scenario (B). We are going to demonstrate that both scenarios are viable in the left-right symmetric model to account for the cosmological baryon number asymmetry and the neutrino mixing data.

The remaining part of this paper is organized as follows. In section II, we diagonalize the FL texture and describe the picture of leptogenesis as the preliminaries. Sections III and IV are devoted to the details of scenarios (A) and (B), respectively. The consequences of both scenarios on neutrino mixing and leptogenesis are also illustrated in these two sections. A summary of our main results is given in section V.

## II. PRELIMINARIES

In this section, we first describe a generic diagonalization of the FL texture and then outline a couple of basic formulas to be used for the calculation of leptogenesis.

### A. Diagonalization of $\mathcal{F}$

The symmetric matrix  $\mathcal{F}$  in Eq. (5) can be diagonalized by the transformation  $U^\dagger \mathcal{F} U^* = \text{Diag}\{f_1, f_2, f_3\}$ , where  $U$  is unitary and  $f_i$  (for  $i = 1, 2, 3$ ) are real and positive. The special texture of  $\mathcal{F}$  guarantees  $U$  to take the form

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos \theta & 0 & \sin \theta e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin \theta e^{i\delta} & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}, \quad (6)$$

where the leading term is just the tri-bimaximal mixing pattern [19], and the parameters  $\theta$  and  $\delta$  are given by

$$\begin{aligned} \tan 2\theta &= \frac{2\sqrt{(|A|^2 + |B|^2)|C|^2 + 2\text{Re}(A^*B^*C^2)}}{|B|^2 - |A|^2}, \\ \tan \delta &= -\frac{\text{Im}[a^*(b-c) + b^*c]}{\text{Re}[(a^* + d^*)(b-c)] + |b|^2 - |c|^2}, \end{aligned} \quad (7)$$

together with  $A = 3(b+c)/2 + d$ ,  $B = 2a + (b+c)/2 + d$  and  $C = -\sqrt{3}(b-c)/2$  [17]. Furthermore, we obtain

$$\begin{aligned} f_1 &= |A - C \tan \theta e^{-i\delta}|, \\ f_2 &= |d|, \\ f_3 &= |B + C \tan \theta e^{+i\delta}|; \end{aligned} \quad (8)$$

and

$$\begin{aligned} \phi_1 &= \frac{1}{2} \arg(A - C \tan \theta e^{-i\delta}), \\ \phi_2 &= \frac{1}{2} \arg(d), \\ \phi_3 &= \frac{1}{2} \arg(B + C \tan \theta e^{+i\delta}). \end{aligned} \quad (9)$$

Without loss of generality, one may rotate away  $\phi_3$  through the redefinitions  $\rho \equiv \phi_1 - \phi_3$  and  $\sigma \equiv \phi_2 - \phi_3$ . Then  $U$  contains three non-trivial phase parameters:  $\delta$ ,  $\rho$  and  $\sigma$ .

Since  $M_L = v_L \mathcal{F}$  and  $M_R = v_R \mathcal{F}$  hold in the left-right symmetric model, their mass eigenvalues can be given in terms of  $f_1$ ,  $f_2$  and  $f_3$  obtained in Eq. (8). For example, three heavy right-handed Majorana neutrino masses are simply  $M_i = v_R f_i$  (for  $i = 1, 2, 3$ ).

## B. Leptogenesis

For simplicity, let us assume that three heavy right-handed Majorana neutrinos have a normal mass hierarchy (i.e.,  $M_1 < M_2 < M_3$ ) and their masses are much smaller than the masses of Higgs triplets. In this case only the CP-violating asymmetry generated from the out-of-equilibrium decay of the lightest right-handed Majorana neutrino can survive and contribute to the thermal leptogenesis [11]. Such a CP-violating asymmetry, denoted as  $\varepsilon_1$ , arises from the interference between the tree-level and one-loop decay amplitudes. In the basis where  $M_R$  is diagonal and real,  $\varepsilon_1$  is given by  $\varepsilon_1 = \varepsilon_1^{(1)} + \varepsilon_1^{(2)}$  with

$$\begin{aligned}\varepsilon_1^{(1)} &= \frac{1}{8\pi v^2} \sum_{j \neq 1} \frac{\text{Im} \left[ \left( \hat{M}_D^\dagger \hat{M}_D \right)_{1j}^2 \right]}{\left( \hat{M}_D^\dagger \hat{M}_D \right)_{11}} \sqrt{x_j} \left[ \frac{2 - x_j}{1 - x_j} - (1 + x_j) \ln \frac{x_j + 1}{x_j} \right], \\ \varepsilon_1^{(2)} &= \frac{3}{16\pi v^2} M_1 \frac{\text{Im} \left[ \left( \hat{M}_D^\dagger M_L \hat{M}_D^* \right)_{11} \right]}{\left( \hat{M}_D^\dagger \hat{M}_D \right)_{11}},\end{aligned}\tag{10}$$

where  $v \simeq 174$  GeV,  $x_j = M_j^2/M_1^2$ , and  $\hat{M}_D$  is the Dirac neutrino mass matrix defined in the basis where  $M_R$  is diagonal. Note that  $\varepsilon_1^{(1)}$  is the conventional CP-violating term, while  $\varepsilon_1^{(2)}$  is due to the one-loop contribution induced by the Higgs triplets [20].

The CP-violating asymmetry  $\varepsilon_1$  can give rise to a net lepton number asymmetry in the Universe, and the nonperturbative sphaleron interaction may partially convert this lepton number asymmetry into a net baryon number asymmetry [21],

$$\eta_B \equiv \frac{n_B}{n_\gamma} \simeq -0.96 \times 10^{-2} \varepsilon_1 \kappa_1,\tag{11}$$

where  $\kappa_1$  is an efficiency factor measuring the washout effects associated with the out-of-equilibrium decay of the lightest right-handed Majorana neutrino. The value of  $\kappa_1$  can be evaluated from the following analytical approximation [21]:  $\kappa_1 = \kappa^-(K_1) + \kappa^+(K_1)$ , where

$$\begin{aligned}\kappa^-(K_1) &= -2e^{-2N(K_1)/3} \left[ e^{2\bar{N}(K_1)/3} - 1 \right], \\ \kappa^+(K_1) &= \frac{2}{z_B(K_1)K_1} \left[ 1 - e^{-2z_B(K_1)K_1\bar{N}(K_1)/3} \right],\end{aligned}\tag{12}$$

with

$$\begin{aligned}\bar{N}(K_1) &= \frac{N(K_1)}{\left( 1 + \sqrt{\frac{N(K_1)}{N_{\text{eq}}}} \right)^2}, \\ z_B(K_1) &\simeq 1 + \frac{1}{2} \ln \left\{ 1 + \frac{\pi K_1^2}{1024} \left[ \ln \left( \frac{3125\pi K_1^2}{1024} \right) \right]^5 \right\}.\end{aligned}\tag{13}$$

Here  $K_1$  is the ratio of the total decay width of the lightest right-handed Majorana neutrino to the expansion rate of the Universe at temperature  $T = M_1$  [21]. One usually expresses  $K_1$  as  $K_1 = \tilde{m}_1/m_*$ , where  $\tilde{m}_1 = (\hat{M}_D^\dagger \hat{M}_D)_{11}/M_1$  denotes the effective (seesaw) neutrino

mass, and  $m_* \simeq 1.08 \times 10^{-3}$  eV stands for the equilibrium neutrino mass. In addition,  $\overline{N}(K_1)$  represents the number density of the lightest right-handed Majorana neutrino, which interpolates between the maximal number densities  $N_{\text{eq}} = 3/4$  and  $N(K_1) = 9\pi K_1/16$  for strong and weak washout regions, respectively.

Note that the formulas listed above are only valid for the flavor-independent leptogenesis mechanism. The flavor-dependent effects will become relevant if thermal leptogenesis takes place at temperatures below  $M_1 \sim 10^{12}$  GeV [22,23]. We shall assume  $M_1 > 10^{12}$  GeV for two phenomenological scenarios to be discussed in the subsequent sections, such that flavor effects on leptogenesis can be safely neglected.

### III. SCENARIO (A)

Scenario (A) is based on three assumptions: (1)  $\kappa' \sim 0$  in Eq. (3), such that  $M_e \simeq \kappa \tilde{Y}_l$  and  $M_D \simeq \kappa Y_l$  (with  $\kappa \simeq v \simeq 174$  GeV) are both symmetric; (2)  $Y_l = \tilde{Y}_l$  is diagonal, or equivalently  $M_D = v \text{Diag}\{y_e, y_\mu, y_\tau\} = \text{Diag}\{m_e, m_\mu, m_\tau\}$ ; and (3)  $\mathcal{F}$  takes the FL texture as shown in Eq. (5). The first assumption is consistent with the general expectation  $\kappa' \ll \kappa$  in the left-right symmetric model [24], while the second and third ones are purely phenomenological. In these assumptions, the effective neutrino mass matrix  $M_\nu$  in Eq. (4) can be explicitly expressed as

$$M_\nu = v_L \left[ \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix} + dI \right] - \frac{v^2}{v_R S} \left[ \begin{pmatrix} y_e^2 (\hat{b} + \hat{c}) & -y_e y_\mu \hat{b} & -y_e y_\tau \hat{c} \\ -y_e y_\mu \hat{b} & y_\mu^2 (\hat{a} + \hat{b}) & -y_\mu y_\tau \hat{a} \\ -y_e y_\tau \hat{c} & -y_\mu y_\tau \hat{a} & y_\tau^2 (\hat{a} + \hat{c}) \end{pmatrix} \right. \\ \left. + \frac{1}{d} \begin{pmatrix} y_e^2 & 0 & 0 \\ 0 & y_\mu^2 & 0 \\ 0 & 0 & y_\tau^2 \end{pmatrix} \right], \quad (14)$$

where  $\hat{a}$ ,  $\hat{b}$ ,  $\hat{c}$  and  $S$  are simple functions of the parameters of  $\mathcal{F}$ :

$$S = d \left[ d^2 + 2(a+b+c)d + 3(ab+bc+ac) \right], \\ \hat{a} = \frac{1}{S} [-ad - (ab+bc+ac)], \\ \hat{b} = \frac{1}{S} [-bd - (ab+bc+ac)], \\ \hat{c} = \frac{1}{S} [-cd - (ab+bc+ac)]. \quad (15)$$

Note that the vevs of the Higgs bi-doublet and triplets are related with one another through  $v_L v_R = \gamma v^2$  in the left-right symmetric model [6], where  $\gamma$  depends on the Higgs potential of the model and is usually expected to be  $\mathcal{O}(1)$ . Typically choosing  $v_L \sim 0.1\text{eV}$  [13], we may simplify Eq. (14) and get an approximate expression of  $M_\nu$  as follows:

$$M_\nu \simeq v_L \left[ \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix} + dI - \Delta \begin{pmatrix} 0 & 0 & 0 \\ 0 & r^2 & -r \\ 0 & -r & 1 \end{pmatrix} \right], \quad (16)$$

where  $r \simeq y_\mu/y_\tau$  and  $\Delta \sim y_\tau^2(\hat{a} + d^{-1})/\gamma$ . Because of  $y_\tau^2 \sim 10^{-4}$ , the  $\Delta$ -term in Eq. (16) is strongly suppressed. It is therefore a good approximation to take  $M_\nu \simeq M_L$  (i.e., only the first two terms on the right-hand side of Eq. (16) are kept). In this case,  $M_\nu$  has the FL texture and can be diagonalized by using the unitary matrix  $U$  given in Eq. (6). Three light (left-handed) Majorana neutrino masses turn out to be  $m_i = v_L f_i$  (for  $i = 1, 2, 3$ ), where  $f_i$  can be found from Eq. (8). Comparing  $m_i$  with the masses of three right-handed Majorana neutrinos  $M_i = v_R f_i$ , we immediately arrive at  $M_i/m_i = v_R/v_L$  in scenario (A). Since  $M_1 < M_2 < M_3$  has been required in discussing leptogenesis,  $m_1 < m_2 < m_3$  must hold (i.e., the light Majorana neutrinos have a normal mass hierarchy). Furthermore, three neutrino mixing angles are given as

$$\begin{aligned}\sin \theta_{12} &= \frac{1}{\sqrt{2 + \cos 2\theta}}, \\ \sin \theta_{23} &= \frac{\sqrt{2 + \cos 2\theta} - \sqrt{3} \sin 2\theta \cos \delta}{\sqrt{2(2 + \cos 2\theta)}}, \\ \sin \theta_{13} &= \frac{2}{\sqrt{6}} \sin \theta\end{aligned}\tag{17}$$

in the standard parametrization [5]. The CP-violating phases of  $U$  have been presented in Eqs. (7) and (9), from which one may define  $\rho = \phi_1 - \phi_3$  and  $\sigma = \phi_2 - \phi_3$  as two independent Majorana phases [25]. In view of  $\theta_{13} < 10^\circ$  as constrained by a global analysis of current experimental data [15], we obtain  $\theta < 12.2^\circ$  from Eq. (17). The smallness of  $\theta$  implies that  $\sin \theta_{12} \simeq 1/\sqrt{3}$  (or  $\theta_{12} \simeq 35.3^\circ$ ) and  $\sin \theta_{23} \simeq 1/\sqrt{2}$  (or  $\theta_{23} \simeq 45^\circ$ ) are excellent approximations; i.e.,  $U$  is a nearly tri-bimaximal neutrino mixing pattern and is strongly favored by the solar and atmospheric neutrino oscillation measurements.

Now let us consider leptogenesis in scenario (A). As one can see from Eq. (10), the CP-violating asymmetry  $\varepsilon_1$  depends on the  $(1, 1)$ ,  $(1, 2)$  and  $(1, 3)$  entries of  $\hat{M}_D^\dagger \hat{M}_D$  as well as the  $(1, 1)$  entry of  $\hat{M}_D^\dagger M_L \hat{M}_D^*$ , where  $\hat{M}_D = M_D U^*$  with  $U$  being determined in Eq. (6). A straightforward calculation yields

$$\begin{aligned}(\hat{M}_D^\dagger \hat{M}_D)_{11} &\simeq m_\tau^2 \left( \frac{1}{3} - \frac{1}{6} \cos 2\theta - \frac{1}{2\sqrt{3}} \sin 2\theta \cos \delta \right), \\ (\hat{M}_D^\dagger \hat{M}_D)_{12} &\simeq m_\tau^2 \left( -\frac{1}{3\sqrt{2}} \cos \theta + \frac{1}{\sqrt{6}} \sin \theta e^{i\delta} \right) e^{i(\rho-\sigma)}, \\ (\hat{M}_D^\dagger \hat{M}_D)_{13} &\simeq m_\tau^2 \left( \frac{1}{2\sqrt{3}} \cos^2 \theta - \frac{1}{6} \sin 2\theta e^{i\delta} - \frac{1}{2\sqrt{3}} \sin^2 \theta e^{2i\delta} \right) e^{i\rho};\end{aligned}\tag{18}$$

and

$$\begin{aligned}(\hat{M}_D^\dagger M_L \hat{M}_D^*)_{11} &\simeq m_\tau^2 e^{2i(\phi_1+\phi_3)} \left[ m_1 \left( \frac{1}{6} \cos^2 \theta - \frac{1}{2\sqrt{3}} \sin 2\theta e^{i\delta} + \frac{1}{2} \sin^2 \theta e^{2i\delta} \right)^2 e^{2i\rho} \right. \\ &\quad + m_2 \left( \frac{1}{3\sqrt{2}} \cos \theta - \frac{1}{\sqrt{6}} \sin \theta e^{i\delta} \right)^2 e^{2i\sigma} \\ &\quad \left. + m_3 \left( \frac{1}{2\sqrt{3}} \cos 2\theta + \frac{1}{12} \sin 2\theta e^{-i\delta} - \frac{1}{4} \sin 2\theta e^{i\delta} \right)^2 \right],\end{aligned}\tag{19}$$

where the terms proportional to  $m_e^2$  and those proportional to  $m_\mu^2$  have been omitted by taking account of  $m_e^2 \ll m_\mu^2 \ll m_\tau^2$ . Eqs. (18) and (19) allow us to calculate the CP-violating asymmetry  $\varepsilon_1$  via Eq. (10) and the baryon number asymmetry  $\eta_B$  via Eqs. (11), (12) and (13) in scenario (A).

For simplicity, we only consider a special but interesting parameter space in our numerical analysis. We assume that  $a$ ,  $b$  and  $c$  are real,  $d$  is complex and  $b = c$  holds. The physical roles of different parameters in this simple example are rather clear: (1) real  $a$ ,  $b$  and  $c$  together with  $b = c$  result in the exact tri-bimaximal mixing [19]; (2)  $m_2 \simeq v_L |d|$  fixes the mass scale and spectrum of three left-handed Majorana neutrinos; (3) the phase of  $d$  is the only source of CP violation which leads to non-vanishing  $\varepsilon_1$  and  $\eta_B$ ; and (4) the  $\Delta$ -induced term in Eq. (16) is essentially negligible. We generate the input points of those free parameters by scanning their possible ranges according to a flat random number distribution. Hence the output points will be a clear reflection of the strong constraints, imposed by scenario (A) itself and by current neutrino oscillation data, on relevant parameters. The following experimental data have been taken into account in our calculations:  $30^\circ \leq \theta_{12} \leq 38^\circ$ ,  $36^\circ \leq \theta_{23} \leq 54^\circ$  and  $\theta_{13} < 10^\circ$  as well as  $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (7.2 \cdots 8.9) \times 10^{-5} \text{ eV}^2$  and  $\Delta m_{32}^2 \equiv m_3^2 - m_2^2 = \pm(2.1 \cdots 3.1) \times 10^{-3} \text{ eV}^2$  [15]. We numerically demonstrate that this scenario can successfully account for the cosmological baryon number asymmetry and have no conflict with the neutrino oscillation measurements. Some results are summarized below.

- In FIG. 1 we show the predicted values of  $\eta_B$  changing with  $M_1$ , the mass of the lightest right-handed Majorana neutrino. We find that the observationally-allowed range of  $\eta_B$  (i.e.,  $\eta_B = (6.1 \pm 0.2) \times 10^{-10}$  [10]) can be reproduced from the flavor-independent leptogenesis in the chosen parameter space with  $4.9 \times 10^{12} \text{ GeV} \leq M_1 \leq 7.7 \times 10^{14} \text{ GeV}$ . In particular,  $M_1 \sim 10^{14} \text{ GeV}$  is most favored.
- Taking  $M_1 = 10^{14} \text{ GeV}$  for example, we are able to fix  $a = \pm(0.17 \cdots 0.33)$ ,  $b = c = \pm(0.023 \cdots 0.037)$ ,  $|d| = (0.085 \cdots 0.099)$  and  $\arg(d) = \pm(1.0^\circ \cdots 18.3^\circ)$ . The values of these parameters are not sensitive to the change of  $M_1$  in its allowed range. It is worth remarking that the constraint on  $|d|$  comes mainly from the choice  $v_L \sim 0.1 \text{ eV}$  and the requirement  $M_1 < M_2 < M_3$ , which is equivalent to  $m_1 < m_2 < m_3$ . On the other hand,  $\arg(d)$  is restricted by both  $\eta_B$  and the neutrino oscillation data.
- Given  $v_R = 10^{16} \text{ GeV}$  for instance, the mass spectrum of three light Majorana neutrinos is  $m_1 = (0.55 \cdots 1.0) \times 10^{-3} \text{ eV}$ ,  $m_2 = (8.5 \cdots 9.8) \times 10^{-3} \text{ eV}$  and  $m_3 = (4.2 \cdots 5.8) \times 10^{-2} \text{ eV}$ ; and that of three heavy Majorana neutrinos is  $M_1 = (0.55 \cdots 1.0) \times 10^{14} \text{ GeV}$ ,  $M_2 = (8.5 \cdots 9.8) \times 10^{14} \text{ GeV}$  and  $M_3 = (4.2 \cdots 5.8) \times 10^{15} \text{ GeV}$ . The normal hierarchy of  $m_i$  implies that the effective mass of the neutrinoless double-beta decay  $\langle m \rangle_{ee}$  must be at the  $\mathcal{O}(10^{-3}) \text{ eV}$  level [15], which is far below the present experimental upper bound  $\langle m \rangle_{ee} < 0.35 \text{ eV}$  [5].

Because of  $b = c$  and the smallness of  $\Delta$  taken in our numerical calculations, the neutrino mixing matrix is essentially the tri-bimaximal mixing pattern with a vanishingly small value of  $\theta_{13}$ . Hence there is no observable effect of CP violation in neutrino oscillations. A detailed analysis of the FL texture with  $b \neq c$  can be found in Ref. [17]. Here we make a numerical check about the possible influence of  $b \neq c$  on  $\eta_B$  in scenario (A). We find that the result shown in FIG. 1 is actually not sensitive to the small difference between  $b$  and  $c$ .



#### IV. SCENARIO (B)

Scenario (B) is based on three assumptions: (1)  $\kappa' \sim 0$  in Eq. (3), such that  $M_e \simeq \kappa \tilde{Y}_l$  and  $M_D \simeq \kappa Y_l$  (with  $\kappa \simeq v \simeq 174$  GeV) are both symmetric; (2)  $\tilde{Y}_l$  is diagonal, but  $Y_l$  takes the FL texture and its parameters are all real; and (3)  $\mathcal{F}$  takes the FL texture as given in Eq. (5). The first and second assumptions allow us to write out  $M_D$  as follows:

$$M_D = v \left[ \begin{pmatrix} b' + c' & -b' & -c' \\ -b' & a' + b' & -a' \\ -c' & -a' & a' + c' \end{pmatrix} + d' I \right]. \quad (20)$$

As  $a'$ ,  $b'$ ,  $c'$  and  $d'$  are all assumed to be real, it is easy to diagonalize  $M_D$  by using the transformation  $V'^\dagger M_D V'^* = \text{Diag}\{D_1, D_2, D_3\}$ , where

$$V' = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos \theta' & 0 & \sin \theta' \\ 0 & 1 & 0 \\ -\sin \theta' & 0 & \cos \theta' \end{pmatrix}. \quad (21)$$

Similar to  $\theta$  in Eq. (7), the rotation angle  $\theta'$  can also be determined in terms of  $a'$ ,  $b'$ ,  $c'$  and  $d'$ . Taking account of Eqs. (5) and (20), we calculate the effective Majorana neutrino mass matrix  $M_\nu$  by means of the Type-II seesaw formula in Eq. (4). We find that  $M_\nu$  has the FL texture as  $\mathcal{F}$  and  $M_D$  do:

$$M_\nu = v_L \left[ \begin{pmatrix} \tilde{b} + \tilde{c} & -\tilde{b} & -\tilde{c} \\ -\tilde{b} & \tilde{a} + \tilde{b} & -\tilde{a} \\ -\tilde{c} & -\tilde{a} & \tilde{a} + \tilde{c} \end{pmatrix} + \tilde{d} I \right], \quad (22)$$

where

$$\begin{aligned} \tilde{a} &= - \left\{ \gamma d \left[ d^2 + 2(a+b+c)d + 3(bc+ab+ac) \right] \right\}^{-1} \{ a'd(a'+d') [3(b+c)+2d] \\ &\quad + d[a'b'(3c+d) + a'c'(3b+d) - b'c'(3a+d)] - d'd[b'(a-c) + c'(a-b)] \} + a, \\ \tilde{b} &= - \left\{ \gamma d \left[ d^2 + 2(a+b+c)d + 3(bc+ab+ac) \right] \right\}^{-1} \{ b'd(b'+d') [3(a+c)+2d] \\ &\quad + d[a'b'(3c+d) + b'c'(3a+d) - a'c'(3b+d)] - d'd[a'(b-c) + c'(b-a)] \} + b, \\ \tilde{c} &= - \left\{ \gamma d \left[ d^2 + 2(a+b+c)d + 3(bc+ab+ac) \right] \right\}^{-1} \{ c'd(c'+d') [3(a+b)+2d] \\ &\quad + d[a'c'(3b+d) + b'c'(3a+d) - a'b'(3c+d)] - d'd[a'(c-b) + b'(c-a)] \} + c, \\ \tilde{d} &= -\frac{d'^2}{\gamma d} + d. \end{aligned} \quad (23)$$

It is obvious that  $M_\nu$  can be diagonalized by a unitary transformation  $V(\hat{\theta}, \hat{\delta}, \hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3)$  which has the same form as  $U$  given in Eq. (6). One may simply use Eqs. (7), (8) and (9) to determine the angle and phase parameters of  $V$  as well as three neutrino mass eigenvalues  $m_i$ , after the replacements  $a \rightarrow \tilde{a}$ ,  $b \rightarrow \tilde{b}$ ,  $c \rightarrow \tilde{c}$ ,  $d \rightarrow \tilde{d}$  are made.

We proceed to consider leptogenesis in scenario (B). A noteworthy feature of this scenario is that the (1, 2) entry of  $\hat{M}_D^\dagger \hat{M}_D$  vanishes. The non-vanishing (1, 1) and (1, 3) elements of  $\hat{M}_D^\dagger \hat{M}_D$  together with the (1, 1) element of  $\hat{M}_D^\dagger M_L \hat{M}_D^*$  are given by

$$\begin{aligned} (\hat{M}_D^\dagger \hat{M}_D)_{11} &= D_1^2 \cos^2(\theta - \theta') + D_3^2 \sin^2(\theta - \theta') + (D_3^2 - D_1^2) \sin 2\theta \sin 2\theta' \sin^2 \frac{\delta}{2}, \\ (\hat{M}_D^\dagger \hat{M}_D)_{13} &= \frac{1}{2} (D_3^2 - D_1^2) [\sin 2\theta \cos 2\theta' - \sin 2\theta' (\cos^2 \theta - \sin^2 \theta e^{2i\delta})] e^{i\rho}; \end{aligned} \quad (24)$$

and

$$(\hat{M}_D^\dagger M_L \hat{M}_D^*)_{11} = m_1 T_1^2 + m_3 T_3^2, \quad (25)$$

where

$$\begin{aligned} T_1 &= \left[ D_1 (\cos \theta \cos \theta' + \sin \theta \sin \theta' e^{-i\delta})^2 + D_3 (\cos \theta \sin \theta' - \sin \theta \cos \theta' e^{i\delta})^2 \right] e^{2i\phi_1}, \\ T_3 &= \frac{1}{2} \left[ D_1 (-\cos 2\theta \sin 2\theta' + \sin 2\theta \cos^2 \theta' e^{-i\delta} + \sin 2\theta \sin^2 \theta' e^{i\delta}) \right. \\ &\quad \left. + D_3 (\cos 2\theta \sin 2\theta' - \sin 2\theta \cos^2 \theta' e^{i\delta} - \sin 2\theta \sin^2 \theta' e^{-i\delta}) \right] e^{i(\phi_1 + \phi_3)}. \end{aligned} \quad (26)$$

We remark that the CP-violating phases come from  $\mathcal{F}$ . In addition, Eq. (24) shows that the CP-violating asymmetry  $\varepsilon_1^{(1)}$  directly depends on  $\theta$  (from  $\mathcal{F}$ ),  $\theta'$  and  $D_3^2 - D_1^2$  (from  $M_D$ ). If these three quantities are very small,  $\varepsilon_1^{(1)}$  will be strongly suppressed.

Now we carry out a numerical analysis of neutrino mixing and leptogenesis in scenario (B), just for the purpose of illustration. We assume that  $a$ ,  $b$  and  $c$  are real but  $d$  is complex. Furthermore, we assume  $a' = a$  and  $b' = b = c' = c$  in order to simplify the calculations. The only source of CP violation is the phase of  $d$ , similar to the situation in scenario (A). It is easy to see that  $b' = b = c' = c$  leads to  $\theta = \theta' = 0$ , and thus  $\varepsilon_1^{(1)}$  vanishes. Non-zero  $\eta_B$  is attributed to non-zero  $\varepsilon_1^{(2)}$  in this scenario. Our numerical results are consistent with both the observational data on  $\eta_B$  [10] and the experimental data on two neutrino mass-squared differences and three mixing angles [15]. Some comments are in order.

- In FIG. 2 we show the predicted values of  $\eta_B$  changing with  $M_1$ . We see that the observationally-allowed range of  $\eta_B$  can be reproduced from the flavor-independent leptogenesis in the chosen parameter space with  $6 \times 10^{12} \text{ GeV} \leq M_1 \leq 1 \times 10^{16} \text{ GeV}$ , where higher values of  $M_1$  have been cut off.
- Taking  $M_1 = 10^{14} \text{ GeV}$  for example, we arrive at the allowed regions for the parameters of  $\mathcal{F}$  and  $M_D$ :  $a = \pm(0.20 \cdots 0.28)$ ,  $b = c = \pm(0.026 \cdots 0.049)$ ,  $|d| = (0.085 \cdots 0.097)$  and  $\arg(d) = \pm(0.23^\circ \cdots 5.7^\circ)$  together with  $d' = (0 \cdots 0.3)$ .
- As a consequence of  $b' = b = c' = c$ , the neutrino mixing matrix is exactly the tri-bimaximal mixing pattern with  $\theta_{12} = 35.3^\circ$ ,  $\theta_{23} = 45^\circ$  and  $\theta_{13} = 0^\circ$ . Hence there is no CP violation in neutrino oscillations. Given  $v_R = 10^{16} \text{ GeV}$  for instance, the mass spectra of light and heavy Majorana neutrinos are  $m_1 = (5.8 \cdots 9.7) \times 10^{-4} \text{ eV}$ ,  $m_2 = (8.5 \cdots 9.5) \times 10^{-3} \text{ eV}$ ,  $m_3 = (4.2 \cdots 5.8) \times 10^{-2} \text{ eV}$  and  $M_1 = (5.8 \cdots 9.7) \times 10^{13} \text{ GeV}$ ,  $M_2 = (8.8 \cdots 9.7) \times 10^{14} \text{ GeV}$ ,  $M_3 = (4.3 \cdots 6.0) \times 10^{15} \text{ GeV}$ , respectively.

Again, the normal hierarchy of  $m_i$  implies that the effective mass of the neutrinoless double-beta decay  $\langle m \rangle_{ee}$  can only reach the  $\mathcal{O}(10^{-3}) \text{ eV}$  level.

## V. SUMMARY

We have proposed two viable Type-II seesaw scenarios for the neutrino mass matrix in the left-right symmetric model. The most salient feature of our scenarios is that the Higgs triplet Yukawa coupling matrix  $\mathcal{F}$  takes the intriguing Friedberg-Lee texture. In the basis where the charged-lepton mass matrix  $M_e$  is diagonal, the Dirac neutrino mass matrix  $M_D$  has been assumed to be identical to  $M_e$  in scenario (A) and to take the FL texture in scenario (B). We have shown that the nearly tri-bimaximal neutrino mixing pattern, which is particularly favored by current neutrino oscillation data, can be naturally derived from both scenarios. Requiring the lightest right-handed Majorana neutrino mass  $M_1$  to be above  $10^{12}$  GeV, we have demonstrated the parameter space of each scenario in which the cosmological baryon number asymmetry can be interpreted via the flavor-independent leptogenesis mechanism.

It is worth emphasizing that some of the assumptions made for the above Type-II seesaw scenarios are just for the sake of simplicity. Hence they are not demanded in more general cases. For example, one may discuss the flavor-dependent thermal leptogenesis to account for the observed baryon number asymmetry of the Universe by allowing  $M_1$  to be below  $10^{12}$  GeV [23]. One may also allow all the parameters of  $\mathcal{F}$  to be complex and independent, in order to generate an experimentally appreciable value for the smallest neutrino mixing angle  $\theta_{13}$  and to give rise to the observable effect of CP violation in neutrino oscillations. In this sense we conclude that our scenarios, which are associated with the left-right gauge symmetry and its spontaneous breaking as well as the FL flavor symmetry and its explicit breaking, have very rich implications and consequences in neutrino phenomenology.

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# FIGURES

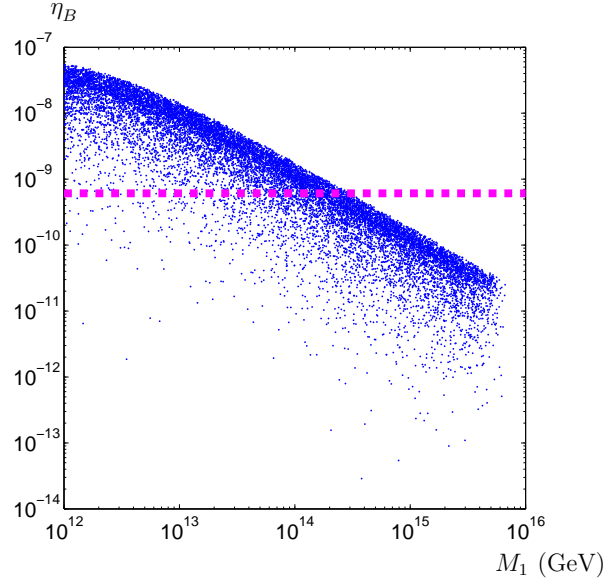


FIG. 1. Illustrative plot for  $\eta_B$  changing with  $M_1$  in scenario (A). Here the dashed band stands for the observationally-allowed range of  $\eta_B$ .

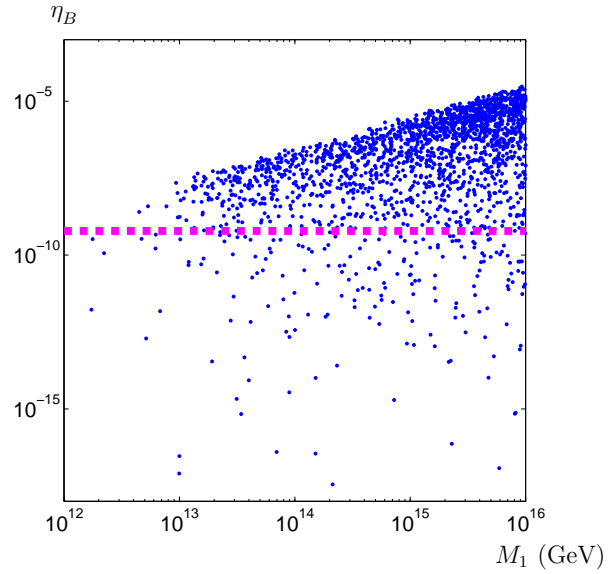


FIG. 2. Illustrative plot for  $\eta_B$  changing with  $M_1$  in scenario (B). Here the dashed band stands for the observationally-allowed range of  $\eta_B$ .